We may also make use of the factorizations (where $\omega^3=1$):

$$A+C=(f-e)(g+f+e),\quad A+\omega C=(f-\omega e)(g+\omega f+\omega^2 e),$$
$$A+\omega^2 C=(f-\omega^2 e)(g+\omega^2 f+\omega e),$$
$$D-B=(f-e)(-h+f+e),\quad \omega D-B=(f-\omega e)(-h+\omega f+\omega^2 e),$$
$$\omega^2 D-B=(f-\omega^2 e)(-h+\omega^2 f+\omega e).$$

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

352. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Mich.

Solve the equations, $x^3=-8y+24\ldots(1)$.
$y^3=-8x+24\ldots(2)$.

Solution by S. LEFSCHETZ, Clark University.

The curves represented by (1) and (2) have nine common points of which three are on the line $x-y=0$, since they are evidently symmetric with respect to it. For these points, then, $x^3+8x-24=0$.

One solution is $x=y=2$; dividing by $x-2$ we obtain, $x^2+2x+12=0$. $\therefore x=y=-1+i\sqrt{11}$, and $x=y=-1-i\sqrt{11}$. To find the solutions, six in number, for which $x\neq y$, we multiply (1) by x, (2) by y, subtract and divide by $x-y$, so that we have,

$$(x+y)(x^2+y^2)=24\ldots(3).$$

Also subtracting (2) from (1), and dividing by $(x-y)$, we have

$$x^2+y^2=8-xy\ldots(4),$$

so that (3) can be written,

$$(x+y)(8-xy)=24\ldots(5),$$

which can also be written,

$$xy(x+y)=8(x+y)-24,$$

and if we add to this last equation after multiplying it by 3, equations (1) and (2), we obtain,

$$x^3+y^3+3xy(x+y)=16(x+y)-24=(x+y)^3.$$

If then $x+y=t$, $xy=u$, we have, $t^3-16t+24=0$, $u=8-24/t$. The cubic can be written,

$$(t-2)(t^2+2t-12)=0.$$

For $t=2$, we have $u=8-\frac{24}{2}=-4$, so that the corresponding values of x or y are roots of $z^2-2z-4=0$; hence one system of values is $x=1+\sqrt{5}$, $y=1-\sqrt{5}$, and another, by permuting x and y.

The other factor of the equation in t has for roots,

$$t_1=-1+\sqrt{13},\quad t_2=-1-\sqrt{13},$$

to which corresponds,

$$u_1=6-2\sqrt{13},\quad u_2=6+2\sqrt{13}.$$

To (t_1, u_1) correspond the roots of $z^2-(-1+\sqrt{13})z+6-2\sqrt{13}=0$, or

$$x=\sqrt{13}-1+\sqrt{[-6\sqrt{(13)}-10]},\quad y=\sqrt{13}-1-\sqrt{[-6\sqrt{(13)}-10]},$$

and the point obtained by permuting x and y.

To have the values of (x, y) corresponding to (t_2, u_2) it is sufficient to change the signs affecting $\sqrt{13}$, in the two preceding ones, and we obtain

$$x=\tfrac{1}{2}[-\sqrt{(13)}-1+\sqrt{[6\sqrt{(13)}-10]},$$
$$y=\tfrac{1}{2}[-\sqrt{(13)}-1-\sqrt{[-6\sqrt{(13)}-10]},$$

and the point obtained by permuting x and y.

Also solved by Jeannette Brooks, Nellie Wood, J. Scheffer, V. M. Spunar, A. H. Holmes, and the Proposer.

353. Proposed by DANIEL KRETH, Oxford, Iowa.

Divide 2940 into two such factors that the square of one factor minus 21 will equal three times the other factor.

Solution by J. K. ELLWOOD, Kansas City, Mo.

Let x be one factor, then $2940/x$ is the other.

By the conditions of the problem, $x^2-21=3\times2940/x=8820/x$.

Whence, $x^3-21x=8820$. Multiplying both sides of the equation by x, $x^4-21x^2=8820x$. Adding $441x^2$ to both sides, $x^4+420x^2=441x^2+8820x$.